3D GALTON BOARD

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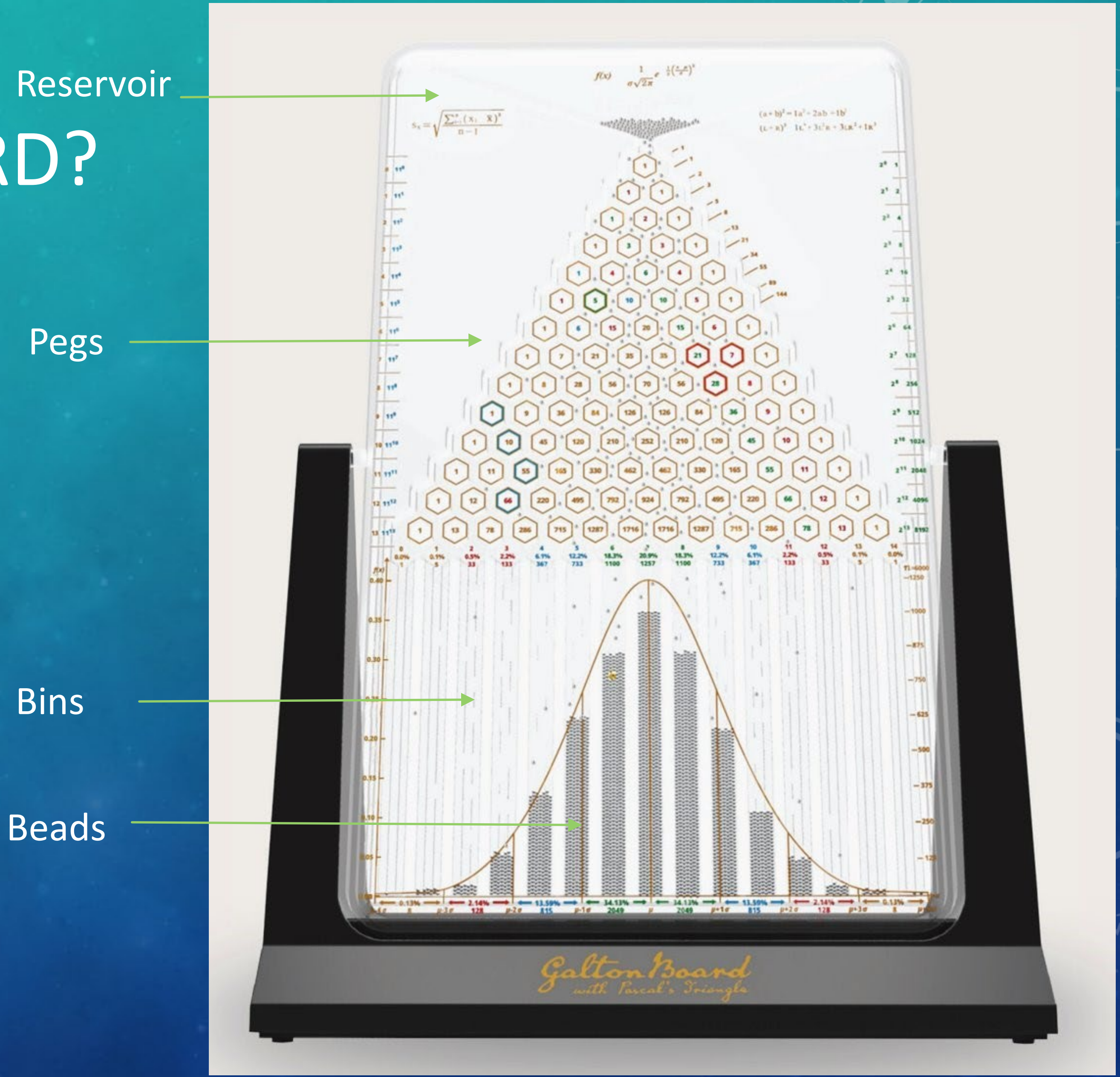
WHAT IS A GALTON BOARD?

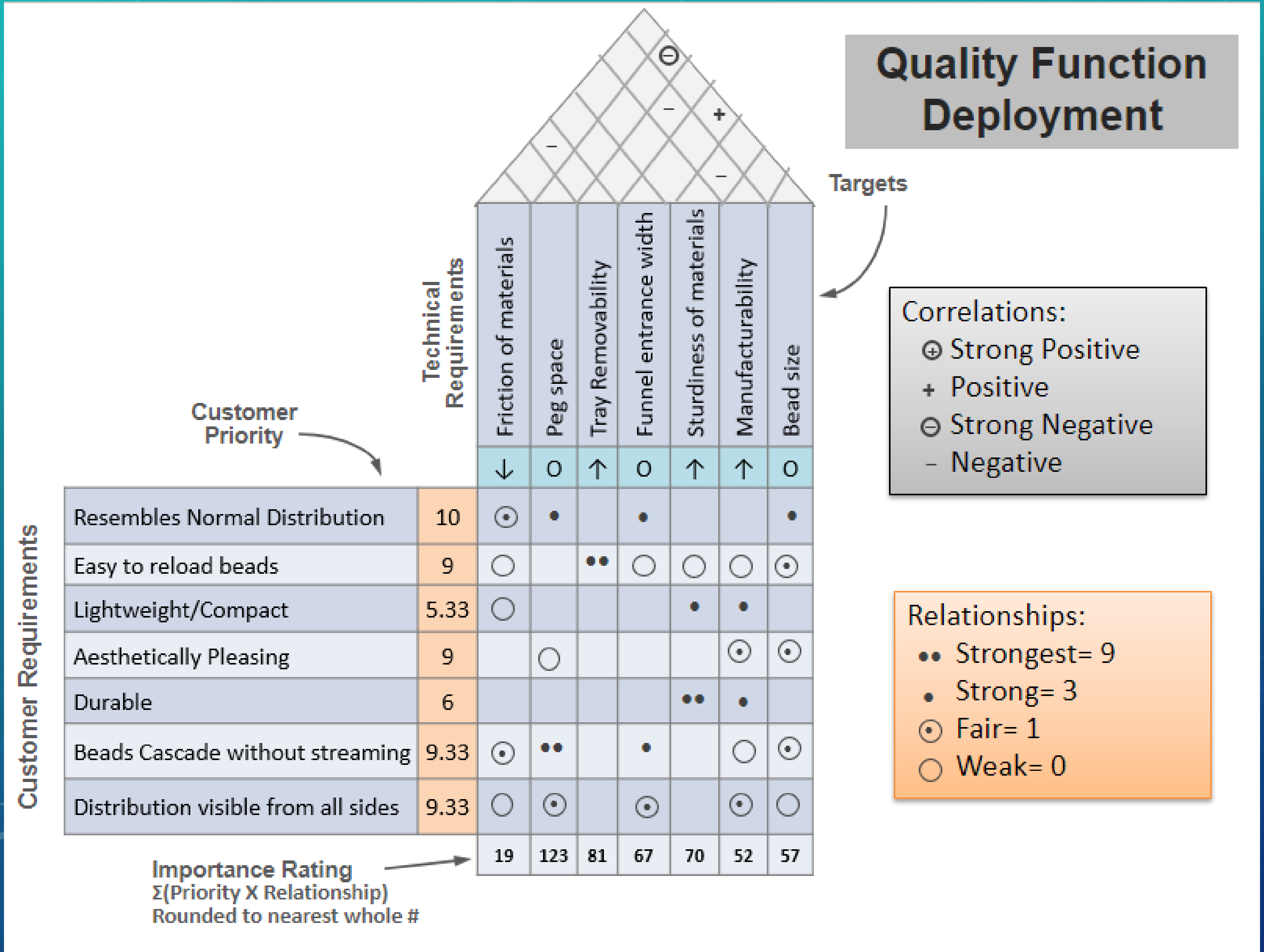
- Invented by Sir Francis Galton in 1876
  - Wanted show that why certain human characteristics varied in an orderly Order out of chaos
- A statistical demonstrator that puts math in motion
  - Creates a normal distribution (bell curve)
- Function resembles y = f (x)
  - x axis shows the variables
  - y axis shows the probability



INTRODUCTION TO THE 3D GALTON BOARD

- What defines a successful 3D Galton board?
  - Teaches statistics
  - Shows a 3D bivariate binomial distribution
  - Good demonstration device to help people learn about the math behind the board
- Function resembles z = f (x , y)
  - Models two independent variables
  - x and y axes show the variables
  - z axis shows the probability

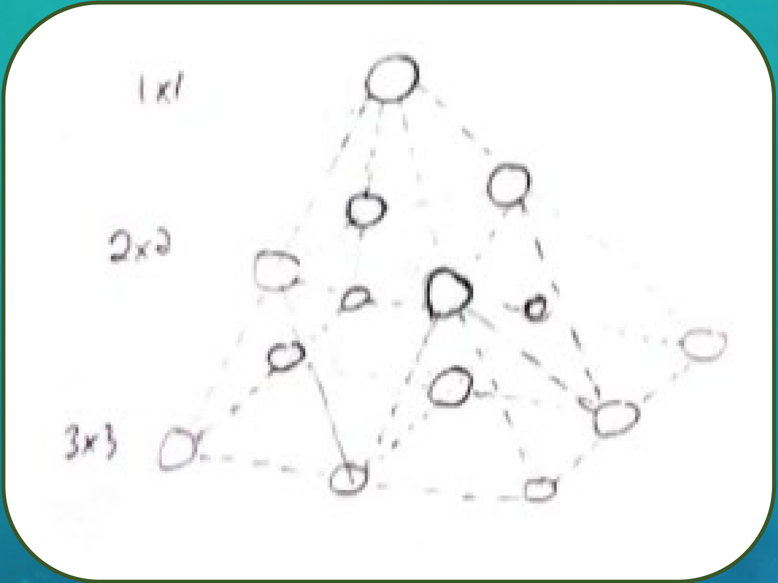
QFD & TECHNICAL REQUIREMENTS



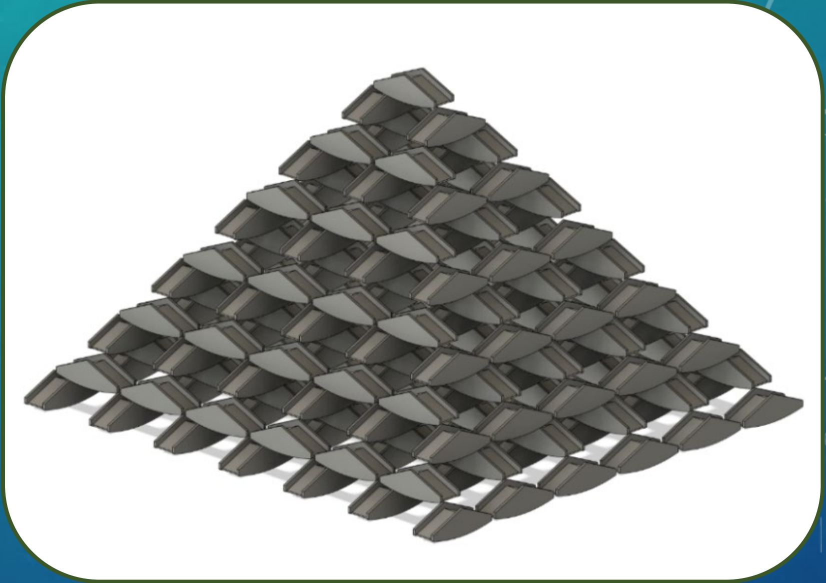
Technical Requirement	Priority Ranking
Peg spacing	1
Tray removability	2
Sturdiness of materials	3
Funnel entrance width	4
Bead size	5
Manufacturability	6
Friction of materials	7

(INITIAL) DESIGN ITERATIONS

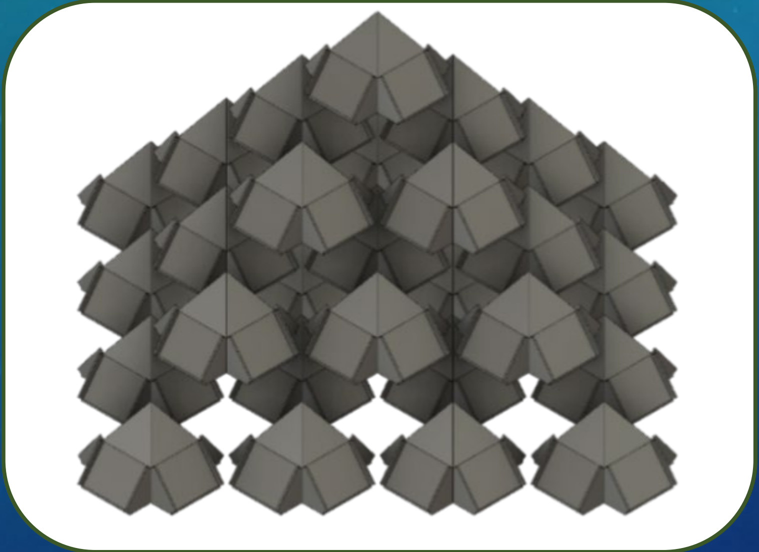
Pyramid of Spheres



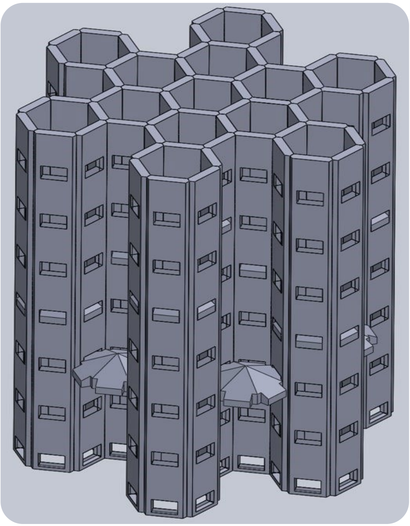
Pyramid of 2-Way Ramps



Pyramid of 4-Way Ramps

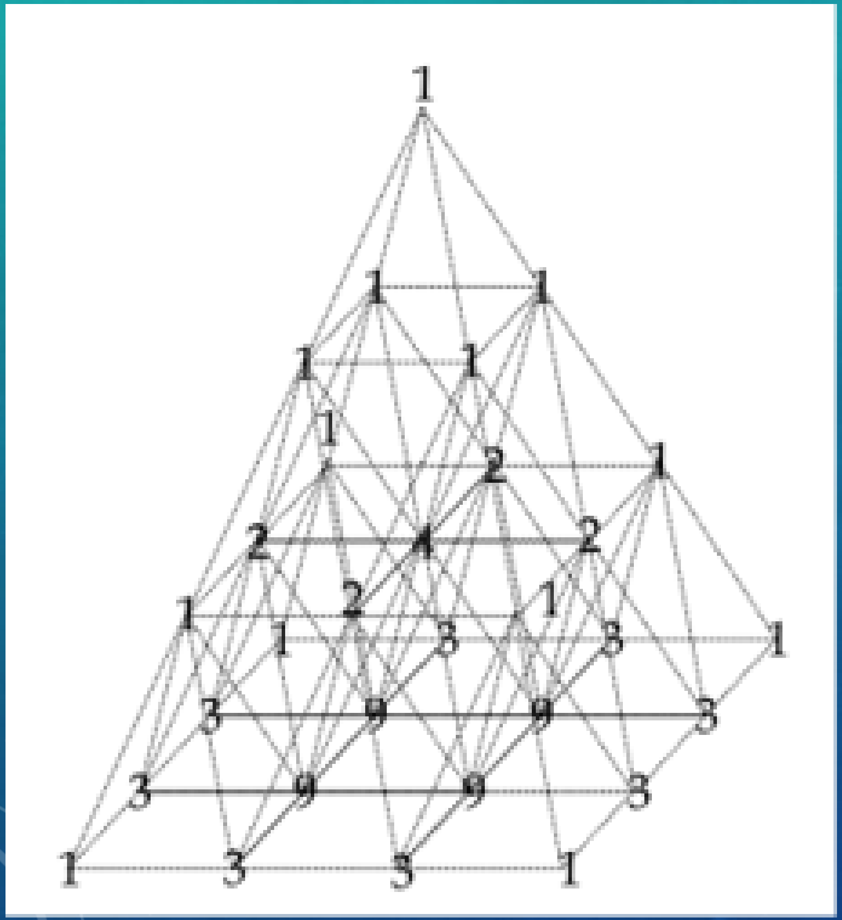


Honeycomb Design

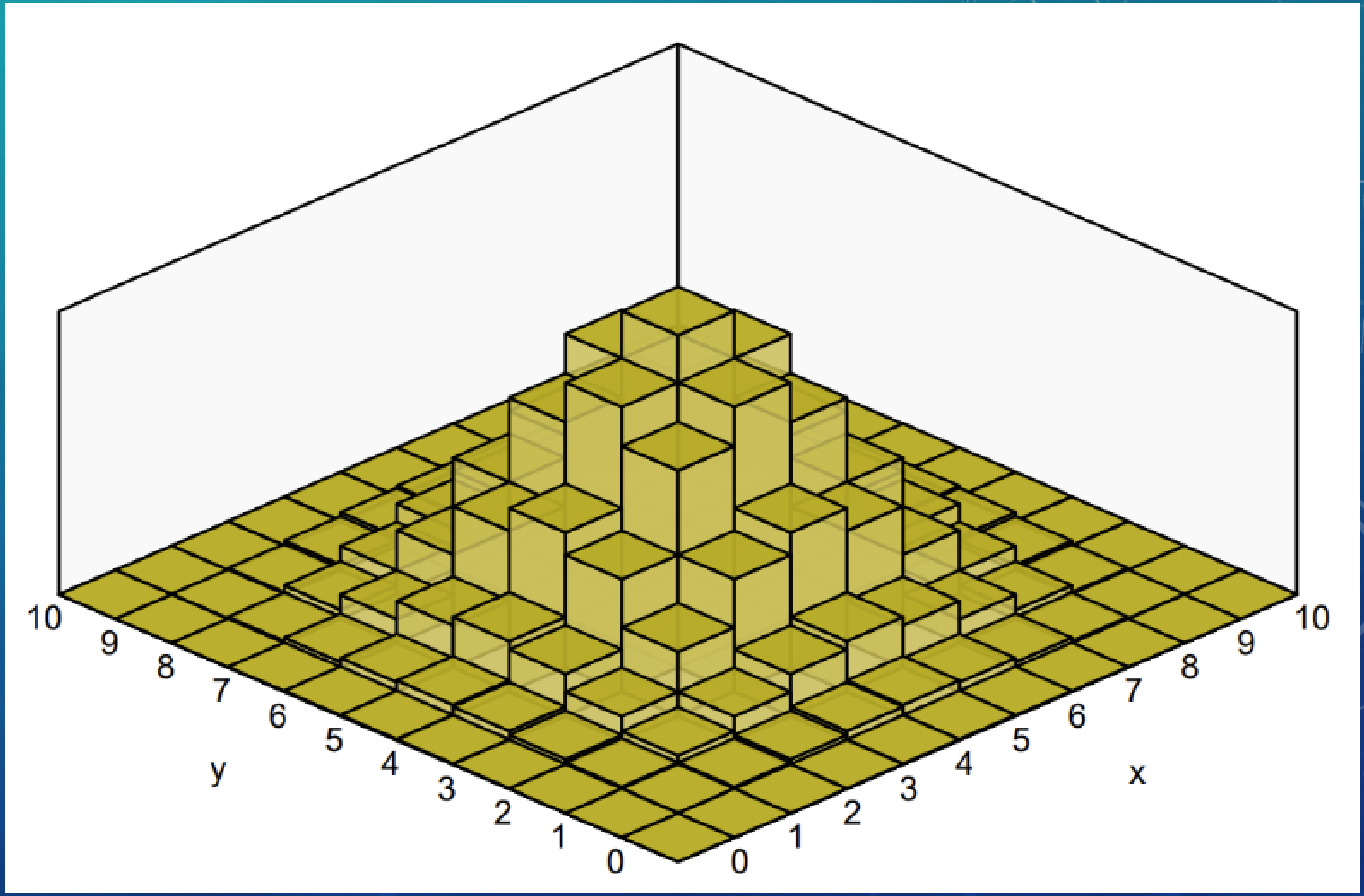


MATHEMATICAL MODELS

Pascal's Square-Based Pyramid



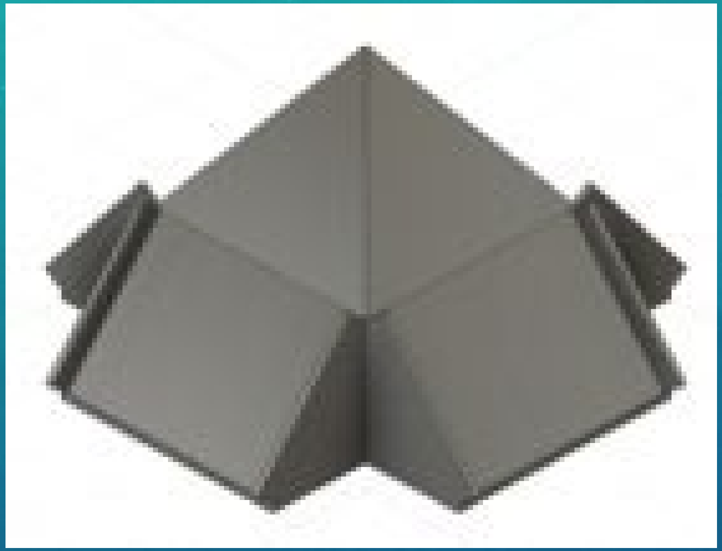
Bivariate Binomial Distribution



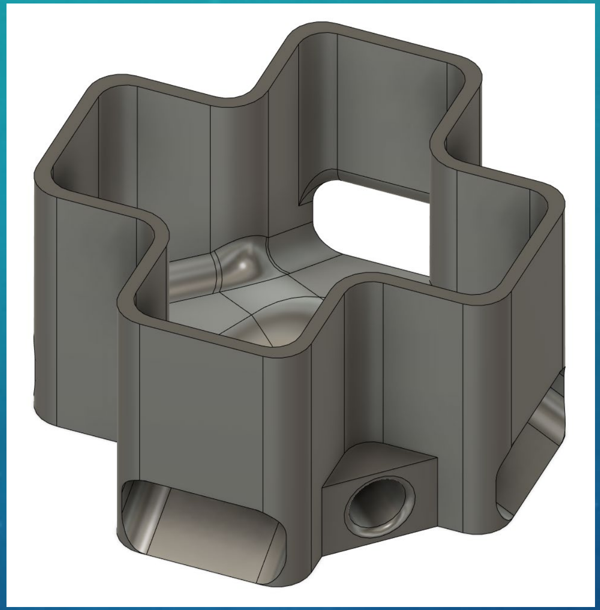
DECISION MATRIX

Requirements	Designs			
	Honeycomb	4-way ramp	2-way ramp	Spheres
Ability to Cascade Beads	3	5	5	1
Ease to make	1	4	5	3
Beads make up 3D distribution	3	5	4	2
Pegs able to be supported	4	5	5	1
No Bead Escape	5	4	4	1
No Bead Jam	2	4	4	4
Cost effectiveness	4	4	4	4
Tolerance Resistance	1	3	3	1
Ranking	3	1	2	4

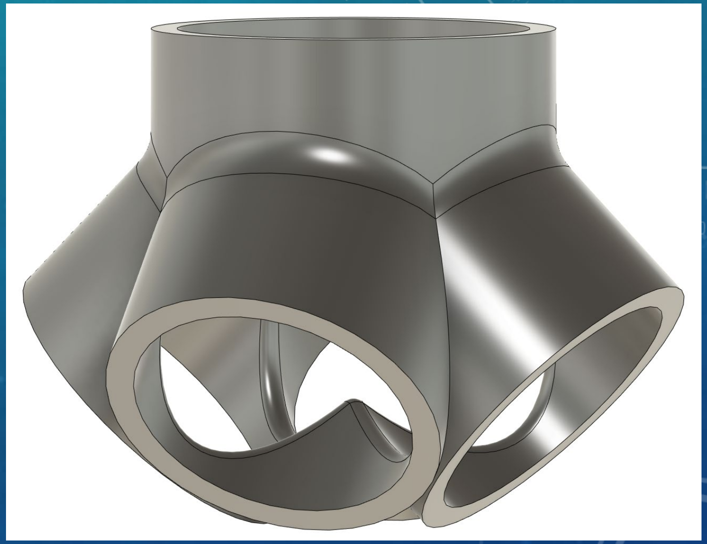
(4-WAY RAMP) DESIGN ITERATIONS



Open 4-Way Ramp



Shielded 4-Way Ramp



Ramp-based sorting manifold

FMEA

Critical failure mode:

Fatigue/wear of manifolds

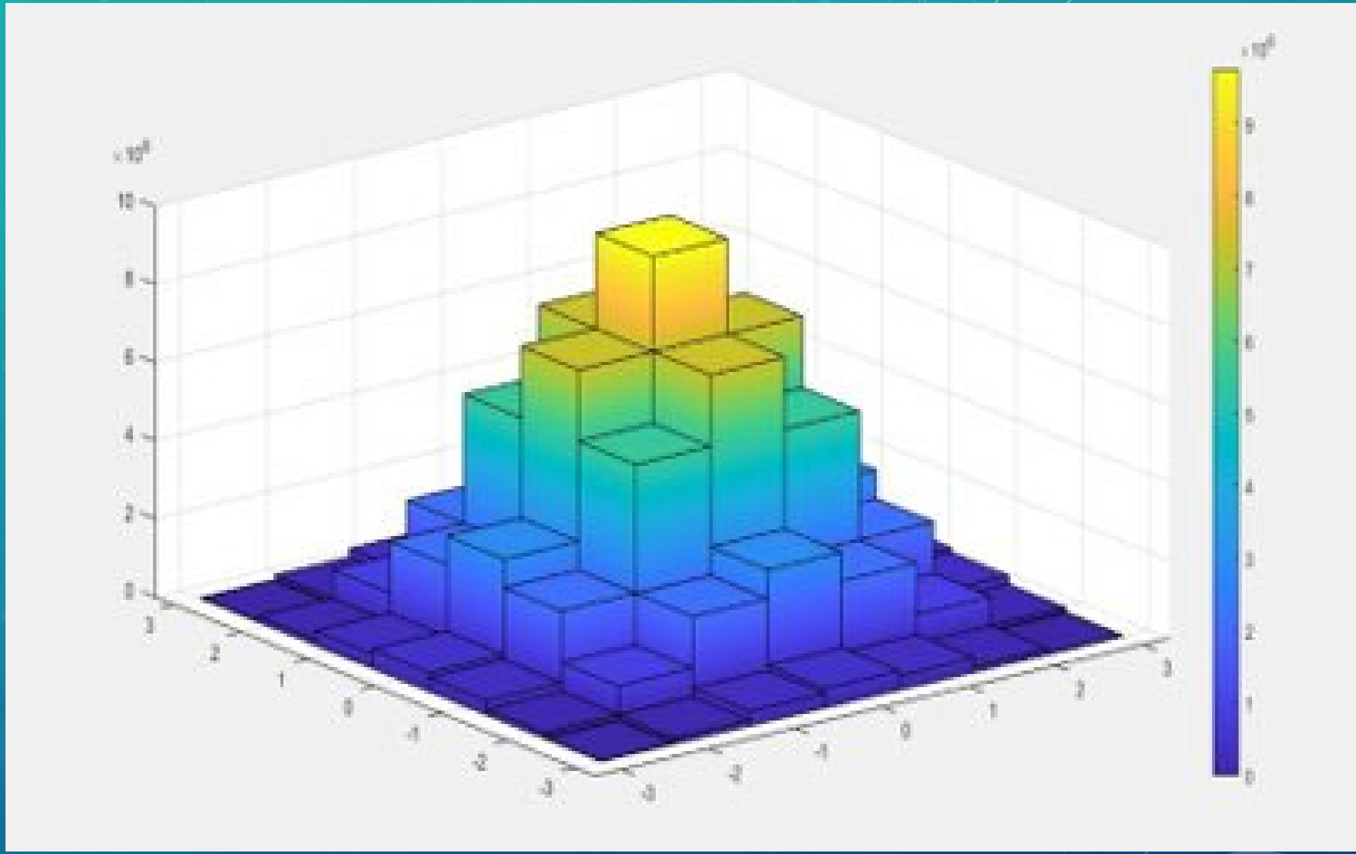
- Severe- Skews final distribution
- Occurrence is uncertain
- Detection is difficult
  - Known properties of material is the only control

Testing suggests that the other failure modes are unlikely to occur

#	Function Affected	Potential Failure Modes	Potential Failure Effects	Potential Causes of Failure	S	O	D	RPN
1	Flow of beads through sorting mechanism	Significant bead jamming	Skewed/incomplete distribution of beads	Beads are too large Adapter/PETG tube diameter too small	7	2	5	70
		Minor bead jamming	Loose beads remain in sorting mechanism after use Distribution is marginally skewed	Sharp corners/bends within sorting mechanism Unpredictable movement of beads due to collisions	5	3	6	90
2	Visibility of bead behavior	User cannot see the complete 3D distribution of beads	Board is less aesthetically pleasing to user Educational value/purpose of board is diminished	High concentration of cylindrical collecting bins	6	2	8	96
		User cannot see the sorting process	Board is less aesthetically pleasing to user	Cloudy/opaque filament used for adapters	6	2	8	96
3	Structural stability of board	Failure at base of bead reservoir	Collapse of bead reservoir/beads spill	Excessive weight of beads Insufficient/improper design of structure	8	2	4	64
		Fatigue/wear of the manifolds	Failure to sort correctly	Small tolerance within manifold structure Continuous use	7	7	7	343
		Failure at tube-manifold connections	Collapse of the sorting structure	Not enough glue Insufficient glue strength	8	2	4	64

SIMULATION RESULTS

- MATLAB CODE
- 100 million beads
- 6 Layers of Pegs
- 7x7 Bin Array
- Drops the Beads Individually
- Matches Bivariate Binomial Model
- 9.7 % of Beads Land in Center Bin
- 0.02 % of Beads Land in Corners



Percentage of Beads That Land in Each Bin

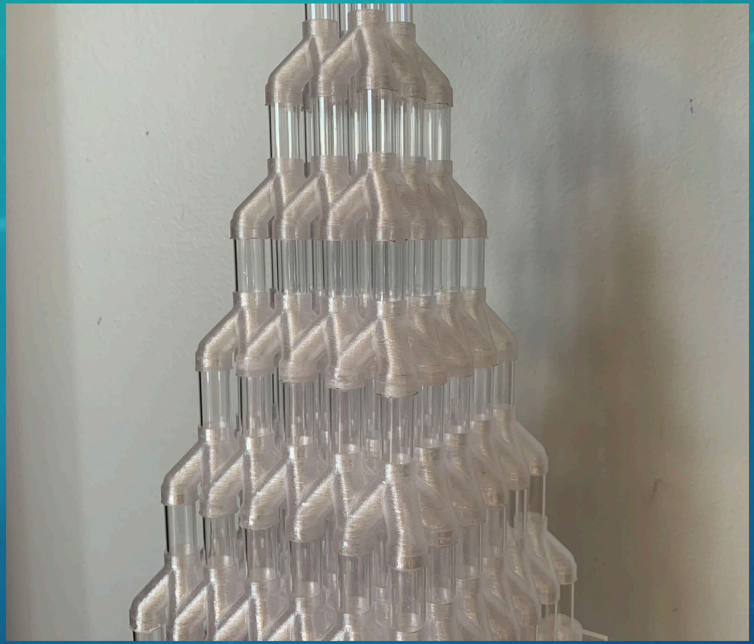
0.0244	0.1468	0.3662	0.4890	0.3671	0.1458	0.0246
0.1464	0.8794	2.1974	2.9301	2.1948	0.8785	0.1462
0.3651	2.1979	5.4922	7.3192	5.4946	2.1999	0.3664
0.4876	2.9326	7.3273	9.7646	7.3266	2.9295	0.4880
0.3663	2.1980	5.4915	7.3239	5.4916	2.1979	0.3656
0.1463	0.8790	2.1979	2.9293	2.1980	0.8787	0.1470
0.0243	0.1458	0.3646	0.4897	0.3652	0.1465	0.0246

FINAL DESIGN/METHODOLOGY

Hopper

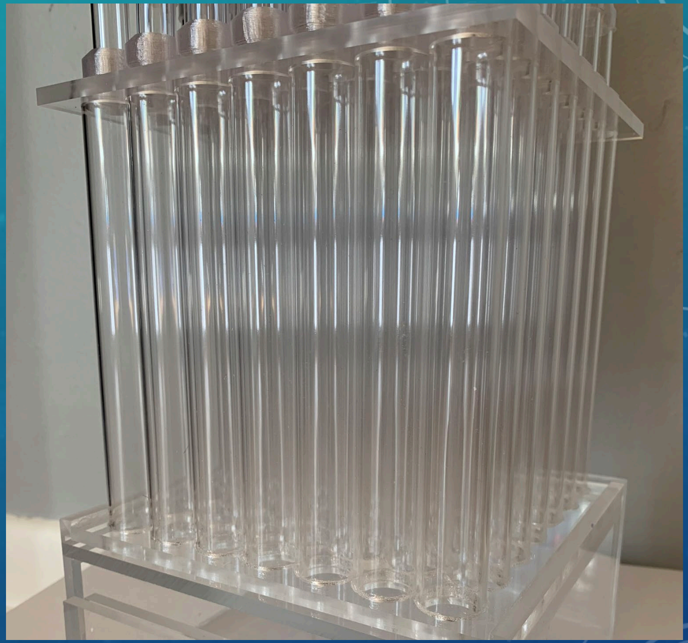
- Side angles and aperture diameter decrease likelihood of bead jamming

Sorting Manifolds



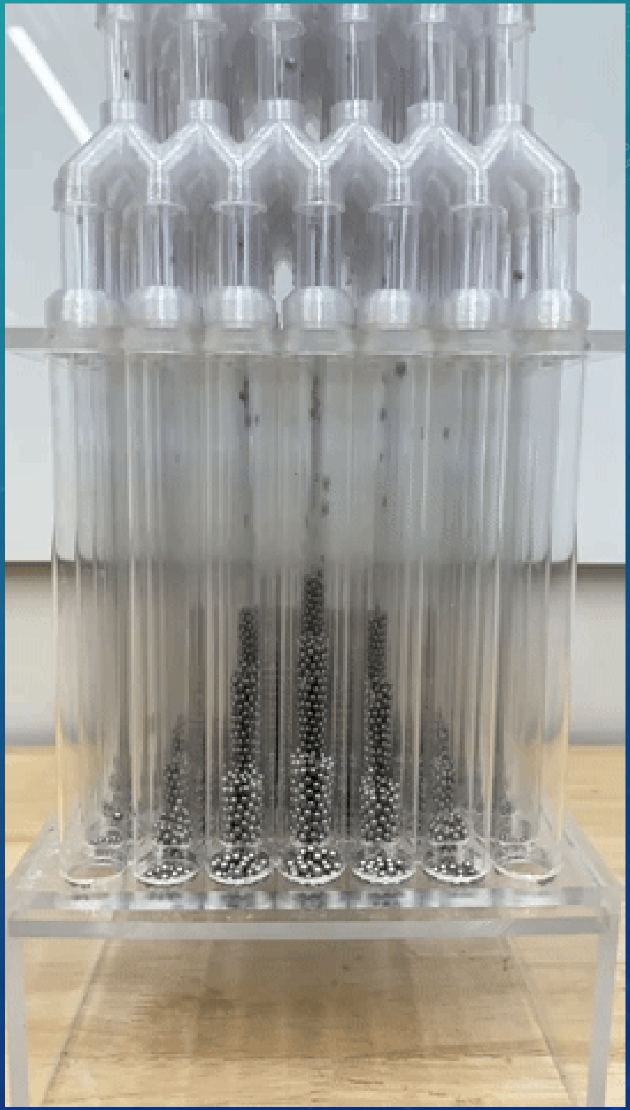
- Completely enclosed
- Tight tolerances & overlap
- Channel diameter and pitch reduce bead jamming chance
- Clarity, clarity, clarity

Display Bins



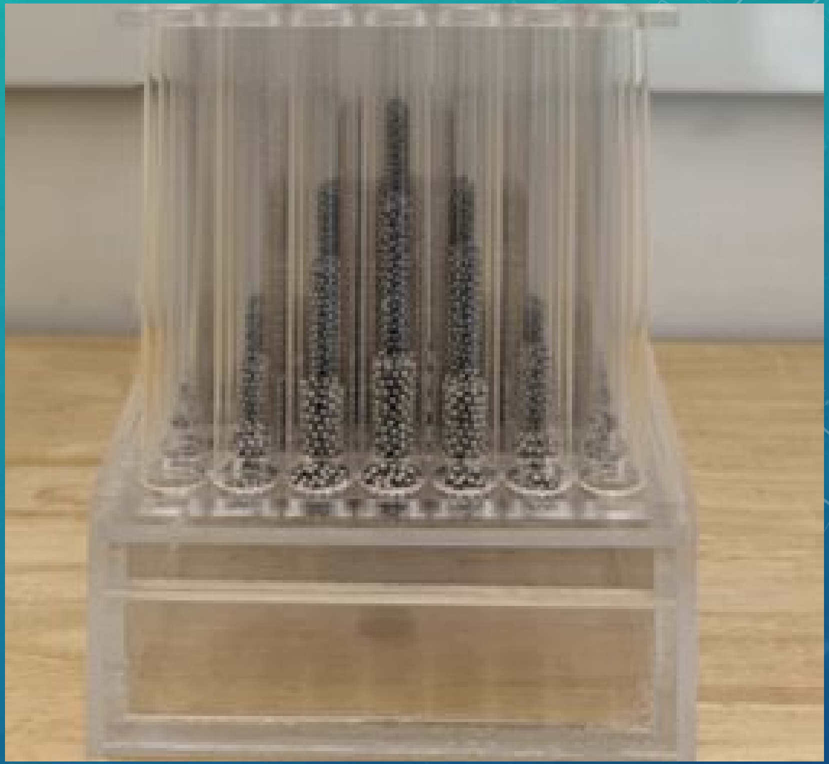
- Bead count vs. lensing
- Quick bead dump and reload

EVERYTHING LOOKS BETTER IN SLOW MOTION



TESTING

- Beads flow Without Jamming
  - Verified by have the beads flow through multiple times.
- Distribution Accuracy
  - Verified the Distribution to the MATLAB simulation.
- Layer Testing
  - Verified with MATLAB simulation.



5 Layers: 500 beads 6X6 bins

MATLAB Code:

0	3	4	13	2	1
4	8	23	25	9	4
6	29	47	48	21	5
1	18	49	54	30	2
2	10	25	27	12	1
0	2	9	4	2	0

Test Result:

0	2	10	5	1	0
3	11	32	20	14	4
5	24	49	49	30	5
1	30	54	42	23	2
3	12	21	26	7	2
0	3	5	2	3	0

RECOMMENDATIONS

- **Manufacturing Processes:** Explore automated machining methods with advanced capabilities to replace manual fabrication and assembly, thereby significantly reducing labor and production time for manufacturing 3D Galton Boards.
- **Improve Component Fabrication:** Investigate faster and cost-effective fabrication methods as alternatives to 3D printing, ensuring reliable production of components without extensive delays.
- **Materials:** Conduct research to identify a reliable and cost-effective supplier for component materials, optimizing the overall cost and quality of the 3D Galton Boards.

CONCLUSION

- **Project Achievement:** Successful completion of the project phases, including design, 3D modeling, and printing, achieving the educational goals and sponsor's expectations. To the best of the team knowledge, it is the first time in history of the world that a physical 3D Galton board has been built.
- **Collaboration:** The collaboration with the sponsor was key in directing the project and ensuring the 3D Galton board's practicality in real-world scenarios. The partnership highlighted the project's relevance and the team's commitment to creating an educational tool that simplifies statistical concepts.
- **Gratitude:** The team expresses gratitude for the guidance provided by the instructor, Prof. John Stang, and the sponsors, Prof. Peter Schubert, Mark Hebner, Philip Poissant, and Art Forster. The project's success is in part a result of their dedication and hard work.

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